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Global Attractivity of a Periodic Ecological Model with m-Predators and n-Preys by "Pure-Delay Type" System

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Abstract—In this paper, based on the comparison theorem and Lyapunov method, we study the following periodic Lotka-Volterra model with m-predators and n-preys by "pure-delay type":

$$\dot{x}_i(t) = x_i(t)\left[b_i(t) - \sum_{k=1}^{n} a_{ik}(t)x_k(t-\tau_{ik}) - \sum_{l=1}^{m} c_{il}(t)y_l(t-\sigma_{il})\right], \qquad i = 1,2,\dots,n,$$
$$\dot{y}_j(t) = y_j(t)\left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t)x_k(t-\xi_{jk}) - \sum_{l=1}^{m} e_{jl}(t)y_l(t-\eta_{jl})\right], \qquad j = 1,2,\dots,m.$$

By proposing a new concept of *generalized uniform M-matrix*, a set of easily verifiable sufficient conditions are obtained for the existence and global attractivity of a unique positive periodic solution of the above model. The obtained results improve and generalize some known results.

Keywords—Predator-prey system, Lyapunov functional, Periodic solutions, Global attractivity.

1. INTRODUCTION

The Lotka-Volterra system is a rudimentary model on mathematical ecology. The asymptotic behavior of the Lotka-Volterra competition system with periodic coefficients has been studied extensively in [1–14]. Some sufficient conditions are obtained for the uniform persistence, existence and uniqueness of the asymptotic stable periodic solution for the Lotka-Volterra competition

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system. Also, some authors obtained some interesting types of explicit solutions such as solitons, positons, and complexitons to the Toda lattice equation and the Lotka-Volterra lattice equation (e.g., see [15] and the references cited therein).

The two-species predator-prey Lotka-Volterra system has been investigated extensively in [16–18], and the references cited therein. Some results were obtained for checking existence of periodic solution and the asymptotic behavior of these systems. But there are few papers considering the multispecies model. Yang and Xu [19] for the first time studied the following periodic system with m-predators and n-preys:

$$\begin{aligned} \dot{x}_i(t) &= x_i(t)\left[b_i(t) - \sum_{k=1}^{n} a_{ik}(t)x_k(t) - \sum_{l=1}^{m} c_{il}(t)y_l(t)\right], \qquad i = 1,2,\dots,n, \\ \dot{y}_j(t) &= y_j(t)\left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t)x_k(t) - \sum_{l=1}^{m} e_{jl}(t)y_l(t)\right], \qquad j = 1,2,\dots,m, \end{aligned} \tag{1.1}$$

where $x_i(t)$ denotes the density of prey species X_i at time t, $y_j(t)$ denotes the density of predator species Y_j at time t; $b_i(t)$, $r_j(t)$, $a_{ik}(t)$, $c_{il}(t)$, $d_{jk}(t)$, and $e_{jl}(t)$, $(i,k = 1,\dots,n;\ j,l = 1,\dots,m)$ are nonnegative continuous periodic functions defined on $t \in (-\infty,+\infty)$. When system (1.1) is autonomous, i.e., when the functions $b_i(t)$, $r_j(t)$, $a_{ik}(t)$, $c_{il}(t)$, $d_{jk}(t)$ and $e_{jl}(t)$, $(i,k = 1,\dots,n;$ $j,l = 1,\dots,m)$ are positive constants b_i, r_j, a_{ik}, c_{il}, d_{jk}, e_{jl}, $(i,k = 1,\dots,n;\ j,l = 1,\dots,m)$, respectively. b_i is the intrinsic growth rate of prey species X_i, r_j is the death rate of the predator species Y_j, a_{ik} measures the amount of competition between the prey species X_i and X_k $(k \neq i,$ $i,k = 1,\dots,n)$, e_{jl} measures the amount of competition between the predator species Y_j and Y_k $(k \neq j,\ j,k = 1,\dots,m)$, and the constant $\tilde{k}_{ij} \triangleq d_{ij}/c_{ij}$ denotes the coefficient in conversing prey species X_i into new individual of predator species Y_j $(i = 1,\dots,n;\ j = 1,\dots,m)$. Some sufficient conditions for existence and global attractivity of a unique positive periodic solution of system (1.1) were obtained. But in their paper, all coefficients are nonnegative. Recently, Zhao and Chen [20] investigated system (1.1) again. In their paper, the intrinsic growth rate of the prey species may be negative while the total intrinsic growth rate in a period are positive. By using the differential inequality theorem and constructing Lyapunov function, some sufficient conditions were obtained for existence and global attractivity of a unique positive periodic solution of (1.1). Recently, Xia *et al.* [21] studied system (1.1) with the almost periodic coefficients. Some sufficient conditions were obtained for existence and global attractivity of a unique positive almost periodic solution of system (1.1).

Seldom did investigate system (1.1) with delay. Wen [22] considered system (1.1) with several delays, that is,

$$\begin{aligned} \dot{x}_i(t) &= x_i(t)\left[b_i(t) - a_i(t)x_i(t) - \sum_{k=1}^{n} a_{ik}(t)x_k(t-\tau_{ik}) - \sum_{l=1}^{m} c_{il}(t)y_l(t-\sigma_{il})\right], \\ &\qquad\qquad i = 1,2,\dots,n, \\ \dot{y}_j(t) &= y_j(t)\left[-r_j(t) - e_j(t)y_j(t) + \sum_{k=1}^{n} d_{jk}(t)x_k(t-\xi_{jk}) - \sum_{l=1}^{m} e_{jl}(t)y_l(t-\eta_{jl})\right], \\ &\qquad\qquad j = 1,2,\dots,m. \end{aligned} \tag{1.2}$$

By means of the comparison theorem and Lyapunov functional, some sufficient conditions were obtained for existence of the global attractivity of a unique positive periodic solution of system (1.2).

In this paper, we consider the periodic Lotka-Volterra model with m-predators and n-preys by "pure-delay type", that is,

$$\begin{aligned}\dot{x}_i(t) &= x_i(t)\left[b_i(t) - \sum_{k=1}^{n} a_{ik}(t)x_k(t-\tau_{ik}) - \sum_{l=1}^{m} c_{il}(t)y_l(t-\sigma_{il})\right], \qquad i = 1,2,\dots,n, \\ \dot{y}_j(t) &= y_j(t)\left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t)x_k(t-\xi_{jk}) - \sum_{l=1}^{m} e_{jl}(t)y_l(t-\eta_{jl})\right], \qquad j = 1,2,\dots,m,\end{aligned} \tag{1.3}$$

where τ_{ik}, σ_{il}, ξ_{il}, η_{jl}, $i,k = 1,\dots,n$; $j,l = 1,\dots,m$ are all nonnegative constants.

Although at first sight, system (1.3) is similar to that of (1.2), however, the results of [22] depend crucially on negative stabilizing feedback terms which act without any time delay, the Lyapunov functionals derived in [22] are not valid for the case (1.3) and so cannot be applied to system (1.3). For the related discussion of "pure-delay type" competition systems, we refer to [13] and the references cited therein. When it comes to multispecies model with m-predators and n-preys, we have not seen any result on the dynamics of the nonautonomous system (1.3) of "pure-delay type". The purpose of this article is to derive a set of sufficient conditions for the existence and global attractivity of a unique positive periodic solution of system (1.3). Our results improve or generalize several previous ones [6,7,9–13,19,20,22].

The method used in this paper is motivated by the works of Gopalsamy and He [13], Zhao and Chen [20], and Burton [23]. The remaining part of this paper is organized as follows. In the next section, we present some definitions and lemmas. In Section 3, by introducing a new concept of *generalized uniform M-matrix* in Section 2, we shall discuss the global attractivity of system (1.3). In Section 4, using the comparison theorem, some sufficient conditions are obtained for the uniform boundedness and ultimately uniform boundedness. Under these conditions, we will establish a criterion for the existence of positive periodic solutions for system (1.3). The method used in this section for the uniform boundedness and ultimately uniform boundedness is much different from that used in [13]. In Section 5, to illustrate the generality of our results, we deduce criteria for some well-known special cases of (1.3), which improve and generalize those available in the literature. Finally, an example is to show how we improve the results.

Throughout this paper, we always assume

$$\inf_{t\in[0,T]} a_{ii}(t) > 0, \qquad \inf_{t\in[0,T]} e_{jj}(t) > 0.$$

Throughout this paper, we shall use the following notations.

- If $f(t)$ is a T-periodic function defined on $(-\infty,+\infty)$, we set

$$\begin{aligned} f^{\iota} &= \inf_{t\in[0,T]} f(t) = \inf_{t\in(-\infty,+\infty)} f(t), \\ f^{\mu} &= \sup_{t\in[0,T]} f(t) = \sup_{t\in(-\infty,+\infty)} f(t), \\ p_i &= \inf_{t\in[0,T]} \frac{b_i(t)}{a_{ii}(t)} \exp\left\{b_i^{\mu}\tau_{ii}\right\}, \\ q_j &= \inf_{t\in[0,T]} \frac{1}{e_{jj}(t)}\left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t)p_k\right] \exp\left\{\left[-r_j^{\iota} + \sum_{k=1}^{n} d_{jk}^{\mu}p_k\right]\eta_{jj}\right\}. \end{aligned}$$

- We always use $i,k = 1,\dots,n$; $j,l = 1,\dots,m$, unless otherwise stated.
- Given $F(t) = (X(t),Y(t))^{\top} = (x_1(t),\dots,x_n(t),y_1(t),\dots,y_m(t))^{\top} \in R^{n+m}$, $G(t) = (U(t), V(t))^{\top} = (u_1(t),\dots,u_n(t),v_1(t),\dots,v_m(t))^{\top} \in R^{n+m}$, we put $F(t) > G(t)$, if $x_i(t) > u_i(t)$, $y_j(t) > v_j(t)$.
- For a given number $\tau > 0$, $\mathbf{C}(C^{+})$ denotes the space of continuous functions mapping the interval $[-\tau,0]$ into R_{+}^{n+m}, $R_{+}^{n+m} = \{x \in R^{n+m};\ x > 0\}$.
- For any $\Phi = (\phi,\psi) = (\phi_1,\dots,\phi_n,\psi_1,\dots,\psi_m) \in \mathbf{C}(C^{+})$, $\|\Phi\| = \sup_{-\tau\le s\le 0}|\Phi(s)|$. $\mathbf{C}_H$ denotes the set of $\Phi \in \mathbf{C}(C^{+})$ for which $\|\Phi\| \le H$.

- For any continuous functions $x(u)$ defined on $-\tau \le u \le A$, $A > 0$ and fixed t, $0 \le t \le A$, the symbol x_t will denote the function $x_t(s) = x(t+s)$ for $-\tau \le s \le 0$.
- Let $\dot{x}(t) = \frac{D^+x(t)}{Dt}$ denote the right-hand derivative of the function $x(t)$, and consider the functional differential equation

$$\dot{x}(t) = F(t, x_t), \tag{1.4}$$

where $F(t,\Phi) \in R^{n+m}$ is defined on $R_+ \times \mathbf{C}$ and continuous in (t,Φ). Let $V(t,\Phi)$ be a functional in t, Φ defined for $t \ge 0$, $\Phi \in \mathbf{C}_H$. The derivative of $V(t,\Phi)$ along the solutions of (1.4) will be denoted by $\dot{V}_{(1.4)}$ and is defined to be

$$\dot{V}_{(1.4)} = \limsup_{\delta\to 0^+} \frac{1}{\delta}\left\{V(t+\delta, x_{t+\delta}(t_0,\Phi)) - V(t, x_t(t_0,\Phi))\right\},$$

where $x_t(t,\Phi)$ is the solution of (1.4) with $x_{t_0}(t_0,\Phi) = \Phi$. A Lyapunov functional for (1.4) can be defined following the notation of Hale and Verduyn Lunel [24].

2. DEFINITIONS AND LEMMAS

Since we are interested in the positive solutions of system (1.3), we assume system (1.3) is supplemented with initial conditions of the form

$$\begin{aligned} x_i(s) &= \phi_i(s) \in \mathbf{C}([-\tau,0], R_+), \qquad \phi_i(0) > 0,\\ y_j(s) &= \psi_j(s) \in \mathbf{C}([-\tau,0], R_+), \qquad \psi_j(0) > 0, \end{aligned} \tag{2.1}$$

where $\tau = \max_{i,k,j,l}\{\tau_{ik},\ \sigma_{il},\ \xi_{jk},\ \eta_{jl}\}$ and $R_+ = (0,+\infty)$. It is easy to see that any solution $F(t) = (X(t),Y(t)) = (x_1(t),\dots,x_n(t),y_1(t),\dots,y_m(t))$ of the initial value problem (1.3),(2.1) exists and satisfies $F(t) > 0$ for $t \ge 0$.

To prove our main results, we shall make the following preparations.

First, we note that properties of M-matrices play a significant role in the study of autonomous mutualistic systems (see [25–27]). In order to study the nonautonomous mutualistic system, Gopalsamy and He [14] generalize the concept of M-matrix to eventually uniform M-matrix (with respect to t). They also used properties of eventually uniform M-matrix to study the competition systems by "pure-delay-type". Now we modify the definition of eventually uniform M-matrix in R^{n+m}.

DEFINITION 2.1. *Let*

$$(G(t))_{(n+m)\times(n+m)} = \begin{pmatrix} (f_{ik}(t))_{n\times n} & (g_{il}(t))_{n\times m} \\ (h_{jk}(t))_{m\times n} & (w_{jl}(t))_{m\times m} \end{pmatrix}$$

be a Z-matrix for all $t \ge 0$. $G(t)$ is called an eventually uniform M-matrix (with respect to t) if there exist positive vectors $\alpha = (s_1,\dots,s_n,\theta_1,\dots,\theta_m)^\top$, $\beta = (\lambda_1,\dots,\lambda_n,\mu_1,\dots,\mu_m)^\top \in R^{n+m}$ and $T_0 \ge 0$ such that

$$G^\top(t)\alpha \ge \beta,$$

for $t \ge T_0$, where $G^\top$ denotes the transpose of G.

In the following, we introduce a new definition of *generalized uniform M-matrix (with respect to t)* which is in some sense, a generalization of the concept of eventually uniform M-matrix (with respect to t).

DEFINITION 2.2. *Let*

$$(G(t))_{(n+m)\times(n+m)} = \begin{pmatrix} (f_{ik}(t))_{n\times n} & (g_{il}(t))_{n\times m} \\ (h_{jk}(t))_{m\times n} & (w_{jl}(t))_{m\times m} \end{pmatrix}$$

be a Z-matrix for all $t \geq 0$. $G(t)$ is called a generalized uniform M-matrix (with respect to t) if there exist a positive constant vector $\alpha = (s_1, \dots, s_n, \theta_1, \dots, \theta_m)^\top$ and a nonnegative function vector $\beta(t) = (\lambda_1(t), \dots, \lambda_n(t), \mu_1(t), \dots, \mu_m(t))^\top \in R^{n+m}$ such that

$$G^\top(t)\alpha \geq \beta(t),$$

for $t \geq 0$, where the functions $\lambda_i(t)$, $\mu_j(t)$ are positive T-periodic functions defined on R and $G^\top$ denotes the transpose of G.

DEFINITION 2.3. *The T-periodic solution $Z(t) = (u_1(t), \dots, u_n(t), v_1(t), \dots, v_m(t))$ is said to be globally attractive, if for every other solution $F(t) = (x_1(t), \dots, x_n(t), y_1(t), \dots, y_m(t))$ of the initial value problem (1.3),(2.1), there are*

$$\lim_{t\to\infty} |u_i(t) - x_i(t)| \to 0, \qquad \lim_{t\to\infty} |v_j(t) - y_j(t)| \to 0.$$

Now we modify the definition of boundedness in [23] or [28], since we are only concerned with the existence of positive periodic solutions of system (1.3).

DEFINITION 2.4. *Solutions of the initial value problem (1.3),(2.1) are said to be uniformly bounded above and below by positive constant vectors at $t = 0$, if for any $\delta^\iota, \delta^\mu \in R^n_+$, $\rho^\iota, \rho^\mu \in R^m_+$ with $0 < \delta^\iota < \delta^\mu$, $0 < \rho^\iota < \rho^\mu$, there exist $\bar{\alpha}$, $M \in R^n_+$, $\bar{\beta}$, $M^* \in R^m_+$ such that, for any $\Phi = (\phi, \psi) \in \mathbf{C}^+(0) = \{\Phi : [-\tau, 0] \to R^{n+m}_+,\ |\Phi \text{ is continuous}\}$ with $\delta^\iota < \phi < \delta^\mu$, $\rho^\iota < \psi < \rho^\mu$, $s \in [-\tau, 0]$, we have $\bar{\alpha} < X(t, \Phi) < M$, $\bar{\beta} < Y(t, \Phi) < M^*$ for $t \geq 0$.*

DEFINITION 2.5. *Solutions of the initial value problem (1.3),(2.1) are said to be uniformly ultimately bounded above and below by positive constant vectors at $t = 0$, if there exist $\alpha, P \in R^n_+$, $\beta, Q \in R^m_+$ with $\alpha < P$, $\beta < Q$ such that, for any $\delta^\iota, \delta^\mu \in R^n_+$, $\rho^\iota, \rho^\mu \in R^m_+$ with $0 < \delta^\iota < \delta^\mu$, $0 < \rho^\iota < \rho^\mu$, there is a $t_1(\delta^\iota, \delta^\mu, \rho^\iota, \rho^\mu)$ for any $\Phi = (\phi, \psi) \in \mathbf{C}^+(0) = \{\Phi : [-\tau, 0] \to R^{n+m}_+,\ |\Phi \text{ is continuous}\}$ with $\delta^\iota < \phi < \delta^\mu$, $\rho^\iota < \psi < \rho^\mu$, $s \in [-\tau, 0]$, we have $\alpha < X(t, \Phi) < P$, $\beta < Y(t, \Phi) < Q$ for $t > t_1(\delta^\iota, \delta^\mu, \rho^\iota, \rho^\mu)$.*

Consider the system

$$\dot{x}(t) = x(t)[b(t) - a(t)x(t)]. \tag{2.2}$$

LEMMA 2.1. *(See [20].) If $\int_0^T b(t)\,dt > 0$, $a(t) > 0$, then system (2.2) has a unique strictly positive ω-periodic solution which is globally asymptotically stable. Moreover, let $x_1(t), x_2(t)$ be the unique strictly positive T-periodic solutions of (2.2) with $b(t) = \hat{b}_1(t), \hat{b}_2(t)$, respectively. If $\hat{b}_1(t) > \hat{b}_2(t)$, then $x_1(0) > x_2(0)$.*

Similar to Lemma 2.1, consider the system

$$\dot{x}_i(t) = x_i(t)\left[b_i(t) - a_{ii}(t)\exp\{-b_i^\mu \tau_{ii}\}\, x_i(t)\right]. \tag{2.3}$$

LEMMA 2.2. *If (H_1): $\int_0^T b_i(t)\,dt > 0$, then system (2.3) has a unique globally attractive positive T-periodic solution, denoted by $x_i^*(t)$.*

Now consider the following system:

$$\begin{aligned} \dot{y}_j(t) = y_j(t) \Bigg[&-r_j(t) + \sum_{k=1}^{n} d_{jk}(t) x_k^*(t - \xi_{jk}) \\ &- e_{jj}(t) \exp\left\{ -\left(-r_j^\iota + \sum_{k=1}^{n} d_{jk}^\mu p_k \right) \eta_{jj} \right\} y_j(t) \Bigg]. \end{aligned} \tag{2.4}$$

LEMMA 2.3. *If* (H_2): $\int_0^T [-r_j(t)+\sum_{k=1}^n d_{jk}(t+\xi_{jk})x_k^*(t)]\,dt > 0$, *then system (2.4) has a unique globally attractive positive* T*-periodic solution, denoted by* $y_j^*(t)$.

PROOF. By the periodicity of $r_j(t)$ and $x_k^*(t)$, we have

$$\int_0^T d_{jk}(t)x_k^*(t-\xi_{jk})\,dt = \int_{-\xi_{jk}}^{T-\xi_{jk}} d_{jk}(s+\xi_{jk})x_k^*(s)\,ds = \int_0^T d_{jk}(s+\xi_{jk})x_k^*(s)\,ds.$$

Hence,

$$\int_0^T \left[-r_j(t)+\sum_{k=1}^n d_{jk}(t)x_k^*(t-\xi_{jk})\right] dt = \int_0^T \left[-r_j(t)+\sum_{k=1}^n d_{jk}(t+\xi_{jk})x_k^*(t)\right] dt > 0.$$

By Lemma 2.1, the assertion of the lemma follows immediately.

Denote

$$B_i(t) = b_i(t) - \sum_{k=1,k\neq i}^n a_{ik}(t)x_k^*(t-\tau_{ik}) - \sum_{l=1}^m c_{il}(t)y_l^*(t-\sigma_{il}),$$

$$A_i(t) = a_{ii}(t)\exp\left\{-\left[b_i^\iota - \sum_{k=1}^n a_{ik}^\mu p_k - \sum_{l=1}^m c_{il}^\mu q_l\right]\tau_{ii}\right\}.$$

Consider system

$$\dot{x}_i(t) = x_i(t)\left[B_i(t) - A_i(t)x_i(t)\right]. \tag{2.5}$$

Similar to the proof of Lemma 2.3, one reaches the following.

LEMMA 2.4. *If*

$$(\mathrm{H}_3)\qquad \int_0^T \left[b_i(t) - \sum_{k=1,k\neq i}^n a_{ik}(t+\tau_{ik})x_k^*(t) - \sum_{l=1}^m c_{il}(t+\sigma_{il})y_l^*(t)\right] dt > 0,$$

system (2.5) has a unique globally attractive positive T*-periodic solution, denoted by* $u_i^0(t)$.

We furthermore assume

$$(\mathrm{H}_4)\qquad \int_0^T \left[-r_j(t)+\sum_{k=1}^n d_{jk}(t)u_k^o(t-\xi_{jk}) - \sum_{l=1,l\neq j}^m e_{jl}(t)y_l^*(t-\eta_{jl})\right] dt > 0.$$

Denote $M = \max_i\{\delta_i^\mu \exp\{b_i^\mu\tau_{ii}\}, \max_{t\in[0,T]} x_i^*(t)\}$ (δ_i^μ defined in Definition 2.4), then from (H_2) and Lemma 2.1, we know

$$\dot{y}_j(t) = y_j(t)\left[-r_j(t)+\sum_{k=1}^n d_{jk}(t)x_k^*(t-\xi_{jk}) - e_{jj}(t)\exp\left\{-\left(-r_j^\iota+\sum_{k=1}^n d_{jk}^\mu M\right)\eta_{jj}\right\}y_j(t)\right]$$

has a unique globally attractive positive T-periodic solution, denoted by $\hat{y}_j^*(t)$.

Denote $M^* \triangleq \max_j\{\rho_j^\mu \exp\{[-r_j^\iota + \sum_{k=1}^n d_{jk}^\mu M]\eta_{jj}\}, \max_{t\in[0,T]} \hat{y}_j^*(t)\}$ (ρ_j^μ defined in Definition 2.4). Then consider the following system:

$$\begin{aligned}\dot{x}_i(t) = x_i(t)&\left[b_i(t) - \sum_{k=1,k\neq i}^n a_{ik}(t)x_k^*(t-\tau_{ik}) - \sum_{l=1}^m c_{il}(t)\hat{y}_l^*(t-\sigma_{il})\right.\\ &\left. - a_{ii}(t)\exp\left\{-\left[b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^*\right]\tau_{ii}\right\}x_i(t)\right].\end{aligned} \tag{2.6}$$

Similar to Lemma 2.2, we have the following lemma.

LEMMA 2.5. *If*

$$\text{(H}_5\text{)}\qquad \int_0^T \left[b_i(t) - \sum_{k=1,k\neq i}^{n} a_{ik}(t+\tau_{ik})x_k^*(t) - \sum_{l=1}^{m} c_{il}(t+\sigma_{il})\hat{y}_l^*(t) \right] dt > 0,$$

then system (2.6) has a unique globally attractive positive T-periodic solution, denoted by $\hat{u}_i^0(t)$.

We furthermore assume

$$\text{(H}_6\text{)}\qquad \int_0^T \left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t+\xi_{jk})\hat{u}_k^0(t) - \sum_{l=1,l\neq j}^{m} e_{jl}(t+\eta_{jl})\hat{y}_l^*(t) \right] dt > 0.$$

LEMMA 2.6. *Suppose Conditions* (H_1)–(H_4) *hold, then all positive solutions* $X(t) = (x_1(t),\dots, x_n(t), y_1(t),\dots, y_n(t))^\top$ *are uniformly ultimately bounded above and there exists a* $T > 0$ *such that*

$$x_i(t) \le M, \quad y_j(t) \le M^*, \qquad \text{for all } t \ge T.$$

For the proof of Lemma 2.6, we refer to the proof of Theorem 4.1 in Section 4.

LEMMA 2.7. *Suppose Conditions* (H_1)–(H_6) *hold, then all positive solutions* $X(t)$ *are uniformly bounded below and and there exist a* $T_0 \ge T$ *and positive constants* $\bar{\alpha}, \bar{\beta}$ *such that*

$$x_i(t) \ge \bar{\alpha}, \quad y_j(t) \ge \bar{\beta}, \qquad \text{for all } t \ge T_0.$$

For the proof of Lemma 2.7, we also refer to the proof of Theorem 4.1 in Section 4.

3. GLOBAL ATTRACTIVITY

Now we are in a position to state our main results.

THEOREM 3.1. *If in addition to* (H_1)–(H_6), *system (1.3),(2.1) also satisfies*

$$B_i(t+\tau_{ii})a_{ii}(t+\tau_{ii})\exp\{b_i^0(t,\tau_{ii})\} < a_{ii}(t),$$
$$H_j(t+\eta_{jj})e_{jj}(t+\eta_{jj})\exp\{r_j^0(t,\eta_{jj})\} < e_{jj}(t),$$

and

$$(G(t))_{(n+m)\times(n+m)} = \begin{pmatrix} (f_{ik}(t))_{n\times n} & (g_{il}(t))_{n\times m} \\ (h_{jk}(t))_{m\times n} & (w_{jl}(t))_{m\times m} \end{pmatrix}$$

is a generalized uniform M-matrix, where $(G(t))_{(n+m)\times(n+m)}$ is defined by

$$B_i(t+\tau_{ik}) = \int_t^{t+\tau_{ii}} A_{ii}(u+\tau_{ik})\,du, \qquad B_i(t+\sigma_{il}) = \int_t^{t+\tau_{ii}} A_{ii}(u+\sigma_{il})\,du,$$

$$H_j(t+\eta_{jl}) = \int_t^{t+\eta_{jj}} E_{jj}(u+\eta_{jl})\,du, \qquad H_j(t+\xi_{jk}) = \int_t^{t+\eta_{jj}} E_{jj}(u+\xi_{jk})\,du.$$

$$b_k^0(t,\tau_{kk}) = \int_{t-\tau_{kk}}^{t} b_k(u)\,du, \qquad r_k^0(t,\eta_{kk}) = \int_{t-\eta_{kk}}^{t} \left[-r_k(u) + \sum_{l=1}^{n} d_{kl}(u)M \right] du.$$

$$f_{ik}(t) = \begin{cases} a_{ii}(t) - B_i(t+\tau_{ii})a_{ii}(t+\tau_{ii})\exp\{b_i^0(t,\tau_{ii})\}, & i = k, \\ -\left[1 + B_i(t+\tau_{ik})\right] a_{ik}(t+\tau_{ik})\exp\{b_k^0(t,\tau_{kk})\}, & i \ne k, \end{cases}$$

$$h_{jk}(t) = -\left[1 + H_j(t+\xi_{jk})\right] d_{jk}(t+\xi_{jk})\exp\{b_k^0(t,\tau_{kk})\},$$

$$g_{il}(t) = -\left[1 + B_i(t+\sigma_{il})\right] c_{il}(t+\sigma_{il})\exp\{r_l^0(t,\eta_{ll})\},$$

$$w_{jl}(t) = \begin{cases} e_{jj}(t) - H_j(t+\eta_{jj})e_{jj}(t+\eta_{jj})\exp\{r_j^0(t,\eta_{jj})\}, & j = k, \\ -\left[1 + H_j(t+\eta_{jl})\right] e_{jl}(t+\eta_{jl})\exp\{r_l^0(t,\eta_{ll})\}, & j \ne k, \end{cases}$$

then system (1.3),(2.1) has a unique globally attractive positive T-periodic solution.

PROOF. Let $F(t) = (x_1(t), \dots, x_n(t), y_1(t), \dots, y_m(t))$ and $Z(t) = (u_1(t), \dots, u_n(t), v_1(t), \dots, v_m(t))$ be any two solutions of the initial value problem (1.3),(2.1). Define

$$\begin{aligned} \tilde{x}_i(t) &= \ln x_i(t), & \tilde{y}_j(t) &= \ln y_j(t), \\ \tilde{u}_i(t) &= \ln u_i(t), & \tilde{v}_j(t) &= \ln v_j(t), \\ W_i(t) &= \tilde{x}_i(t) - \tilde{u}_i(t), & z_j(t) &= \tilde{y}_j(t) - \tilde{v}_j(t). \end{aligned} \tag{3.1}$$

From (3.1) and the first equation of (1.3), we obtain

$$\frac{d}{dt}\left[\tilde{x}_i(t) - \tilde{u}_i(t)\right] = -\sum_{k=1}^{n} a_{ik}(t)\left[e^{\tilde{x}_k(t-\tau_{ik})} - e^{\tilde{u}_k(t-\tau_{ik})}\right] - \sum_{l=1}^{m} c_{il}(t)\left[e^{\tilde{y}_l(t-\sigma_{il})} - e^{\tilde{v}_l(t-\sigma_{il})}\right]. \tag{3.2}$$

Using the mean-valued theorem,

$$e^p - e^q = \int_0^1 \frac{d}{ds}\left[e^{ps}e^{q(1-s)}\right] ds = \left[\int_0^1 \left[e^{ps}e^{q(1-s)}\right] ds\right][p-q]. \tag{3.3}$$

One can write equation (3.2) as follows:

$$\begin{aligned} &\frac{d}{dt}\left[\tilde{x}_i(t) - \tilde{u}_i(t)\right] \\ &= -\sum_{k=1}^{n} a_{ik}(t)\left[\int_0^1 e^{s\tilde{x}_k(t-\tau_{ik})}e^{(1-s)\tilde{u}_k(t-\tau_{ik})}\, ds\right]\left[\tilde{x}_k(t-\tau_{ik}) - \tilde{u}_k(t-\tau_{ik})\right] \\ &\quad - \sum_{l=1}^{m} c_{il}(t)\left[\int_0^1 e^{s\tilde{y}_l(t-\sigma_{il})}e^{(1-s)\tilde{v}_l(t-\sigma_{il})}\, ds\right]\left[\tilde{y}_l(t-\sigma_{il}) - \tilde{v}_l(t-\sigma_{il})\right]. \end{aligned} \tag{3.4}$$

Denote

$$A_{ik}(t) = a_{ik}(t)\left[\int_0^1 e^{s\tilde{x}_k(t-\tau_{ik})}e^{(1-s)\tilde{u}_k(t-\tau_{ik})}\, ds\right], \tag{3.5}$$

$$C_{il}(t) = c_{il}(t)\left[\int_0^1 e^{s\tilde{y}_l(t-\sigma_{il})}e^{(1-s)\tilde{v}_l(t-\sigma_{il})}\, ds\right]. \tag{3.6}$$

Then (3.1) can be rewritten as

$$\begin{aligned} \frac{d}{dt}W_i(t) &= -\sum_{k=1}^{n} A_{ik}(t)W_k(t-\tau_{ik}) - \sum_{l=1}^{m} C_{il}(t)z_l(t-\sigma_{il}) \\ &= -A_{ii}(t)W_i(t) - \sum_{k=1,k\neq i}^{n} A_{ik}(t)W_k(t-\tau_{ik}) \\ &\quad + A_{ii}(t)\int_{t-\tau_{ii}}^{t} \frac{d}{ds}W_i(s)\, ds - \sum_{l=1}^{m} C_{il}(t)z_l(t-\sigma_{il}) \\ &= -A_{ii}(t)W_i(t) - \sum_{k=1,k\neq i}^{n} A_{ik}(t)W_k(t-\tau_{ik}) \\ &\quad - A_{ii}(t)\sum_{k=1}^{n}\int_{t-\tau_{ii}}^{t} A_{ik}(s)W_k(s-\tau_{ik})\, ds \\ &\quad - \sum_{l=1}^{m} C_{il}(t)z_l(t-\sigma_{il}) - A_{ii}(t)\sum_{l=1}^{m}\int_{t-\tau_{ii}}^{t} C_{il}(s)z_l(s-\sigma_{il})\, ds. \end{aligned} \tag{3.7}$$

Let

$$V_{i1}(W)(t) = |W_i(t)|.$$

Then it follows from $A_{ik}(t), C_{il}(t) \geq 0$ that the upper right derivative $\left(\frac{D^+}{Dt}\right) V_{i1}(W)$ of $V_{i1}(W)$ along the solutions of (3.7) is given by

$$\begin{aligned} \frac{D^+}{Dt} V_{i1}(W)(t) \leq -A_{ii}(t)|W_i(t)| + \sum_{k=1,k\neq i}^{n} A_{ik}(t)|W_k(t-\tau_{ik})| \\ + A_{ii}(t) \sum_{k=1}^{n} \int_{t-\tau_{ii}}^{t} A_{ik}(s)|W_k(s-\tau_{ik})|\, ds + \sum_{l=1}^{m} C_{il}(t)|z_l(t-\sigma_{il})| \\ + A_{ii}(t) \sum_{l=1}^{m} \int_{t-\tau_{ii}}^{t} C_{il}(s)|z_l(s-\sigma_{il})|\, ds. \end{aligned} \tag{3.8}$$

Now we define

$$V_i(W)(t) = V_{i1}(W)(t) + V_{i2}(W)(t), \tag{3.9}$$

where $V_{i2}(W)(t)$ is defined by

$$\begin{aligned} V_{i2}(W)(t) = \sum_{k=1,k\neq i}^{n} \int_{t-\tau_{ik}}^{t} A_{ik}(u+\tau_{ik})|W_k(u)|\, du \\ + \sum_{k=1}^{n} \int_{t-\tau_{ii}}^{t} A_{ii}(v+\tau_{ii}) \int_{v}^{t} A_{ik}(u)|W_k(u-\tau_{ik})|\, du\, dv \\ + \sum_{k=1}^{n} \int_{t-\tau_{ik}}^{t} B_i(u+\tau_{ik}) A_{ik}(u+\tau_{ik})|W_k(u)|\, du + \sum_{l=1}^{m} \int_{t-\sigma_{il}}^{t} C_{il}(u+\sigma_{il})|z_l(u)|\, du \\ + \sum_{l=1}^{m} \int_{t-\tau_{ii}}^{t} A_{ii}(v+\tau_{ii}) \int_{v}^{t} C_{il}(u)|z_l(u-\sigma_{il})|\, du\, dv \\ + \sum_{l=1}^{m} \int_{t-\sigma_{il}}^{t} B_i(u+\sigma_{il}) C_{il}(u+\sigma_{il})|z_l(u)|\, du, \end{aligned} \tag{3.10}$$

where

$$B_i(t) = \int_{t-\tau_{ii}}^{t} A_{ii}(u+\tau_{ii})\, du = \int_{t}^{t+\tau_{ii}} A_{ii}(u)\, du. \tag{3.11}$$

It follows from (3.8)–(3.11) that $V_i(W)(t) \geq 0$ and

$$\begin{aligned} \frac{D^+}{Dt} V_i(W)(t) \leq -A_{ii}(t)|W_i(t)| + \sum_{k=1,k\neq i}^{n} A_{ik}(t+\tau_{ik})|W_k(t)| \\ + \sum_{k=1}^{n} B_i(t+\tau_{ik}) A_{ik}(t+\tau_{ik})|W_k(t)| + \sum_{l=1}^{m} C_{il}(t+\sigma_{il})|z_l(t)| \\ + \sum_{l=1}^{m} B_i(t+\sigma_{il}) C_{il}(t+\sigma_{il})|z_l(t)|. \end{aligned} \tag{3.12}$$

From (3.1) and the second equation of (1.3), we obtain

$$\frac{d}{dt}\left[\tilde{y}_j(t) - \tilde{v}_j(t)\right] = \sum_{k=1}^{n} d_{jk}(t) \left[e^{\tilde{x}_k(t-\xi_{jk})} - e^{\tilde{u}_k(t-\xi_{jk})}\right] - \sum_{l=1}^{m} e_{jl}(t) \left[e^{\tilde{y}_l(t-\eta_{jl})} - e^{\tilde{v}_l(t-\eta_{jl})}\right]. \tag{3.13}$$

Using (3.3), one can write equations (3.13) as follows:

$$\begin{aligned}&\frac{d}{dt}\left[\tilde{y}_j(t)-\tilde{v}_j(t)\right]\\&=\sum_{k=1}^{n} d_{jk}(t)\left[\int_0^1 e^{s\tilde{x}_k(t-\xi_{jk})}e^{(1-s)\tilde{u}_k(t-\xi_{jk})}\,ds\right]\left[\tilde{x}_k(t-\xi_{jk})-\tilde{u}_k(t-\xi_{jk})\right]\\&-\sum_{l=1}^{m} e_{jl}(t)\left[\int_0^1 e^{s\tilde{y}_l(t-\eta_{ik})}e^{(1-s)\tilde{v}_l(t-\eta_{jl})}\,ds\right]\left[\tilde{y}_l(t-\eta_{jl})-\tilde{v}_l(t-\eta_{jl})\right].\end{aligned}\tag{3.14}$$

Denote

$$D_{jk}(t)=d_{jk}(t)\left[\int_0^1 e^{s\tilde{x}_k(t-\xi_{jk})}e^{(1-s)\tilde{u}_k(t-\xi_{jk})}ds\right],\tag{3.15}$$

$$E_{jl}(t)=e_{jl}(t)\left[\int_0^1 e^{s\tilde{y}_l(t-\eta_{jl})}e^{(1-s)\tilde{v}_l(t-\eta_{jl})}ds\right].\tag{3.16}$$

Then (3.13) can be rewritten as

$$\begin{aligned}\frac{d}{dt}z_j(t)&=\sum_{k=1}^{n}D_{jk}(t)W_k(t-\xi_{jk})-\sum_{l=1}^{m}E_{jl}(t)z_l(t-\eta_{jl})\\&=-E_{jj}(t)z_j(t)-\sum_{l=1,l\neq j}^{m}E_{jl}(t)z_l(t-\eta_{jl})\\&+E_{jj}(t)\int_{t-\eta_{jj}}^{t}\frac{d}{ds}z_j(s)\,ds+\sum_{k=1}^{n}D_{jk}(t)W_k(t-\xi_{jk})\\&=-E_{jj}(t)z_i(t)-\sum_{l=1,l\neq j}^{m}E_{jl}(t)z_l(t-\eta_{ik})\\&-E_{jj}(t)\sum_{l=1}^{m}\int_{t-\eta_{jj}}^{t}E_{jl}(s)z_l(s-\eta_{jl})\,ds+\sum_{k=1}^{n}D_{jk}(t)W_k(t-\xi_{jk})\\&+E_{jj}(t)\sum_{k=1}^{n}\int_{t-\eta_{jj}}^{t}D_{jk}(s)W_k(s-\xi_{jk})\,ds.\end{aligned}\tag{3.17}$$

Let

$$V_{j1}(z)(t)=|z_j(t)|.$$

Then it follows from $E_{jl}(t), D_{jk}(t)\geq 0$ that the upper right derivative $\left(\frac{D^+}{Dt}\right)V_{j1}(z)$ of $V_{j1}(z)$ along the solutions of (3.17) is given by

$$\begin{aligned}\frac{D^+}{Dt}V_{j1}(z)(t)&\leq -E_{jj}(t)|z_j(t)|+\sum_{l=1,l\neq j}^{m}E_{jl}(t)|z_l(t-\eta_{jl})|\\&+E_{jj}(t)\sum_{l=1}^{m}\int_{t-\eta_{jj}}^{t}E_{jl}(s)|z_l(s-\eta_{jl})|\,ds+\sum_{k=1}^{n}D_{jk}(t)|W_k(t-\xi_{jk})|\\&+E_{jj}(t)\sum_{k=1}^{n}\int_{t-\eta_{jj}}^{t}D_{jk}(s)|W_k(s-\xi_{jk})|\,ds.\end{aligned}\tag{3.18}$$

Now we define

$$V_i(z)(t)=V_{j1}(z)(t)+V_{j2}(z)(t),\tag{3.19}$$

where $V_{j2}(z)(t)$ is defined by

$$
\begin{aligned}
V_{j2}(z)(t) &= \sum_{l=1,l\neq j}^{m} \int_{t-\eta_{jl}}^{t} E_{jl}(u+\eta_{jl})|z_l(u)|\,du \\
&+\sum_{l=1}^{m} \int_{t-\eta_{jj}}^{t} E_{jj}(v+\eta_{jj}) \int_{v}^{t} E_{jl}(u)|z_l(u-\eta_{jl})|\,du\,dv \\
&+\sum_{l=1}^{m} \int_{t-\eta_{jl}}^{t} H_j(u+\eta_{jl})E_{jl}(u+\eta_{jl})|z_l(u)|\,du \\
&+\sum_{k=1}^{n} \int_{t-\xi_{jk}}^{t} D_{jk}(u+\xi_{jk})|W_k(u)|\,du \\
&+\sum_{k=1}^{n} \int_{t-\eta_{jj}}^{t} E_{jj}(v+\eta_{jj}) \int_{v}^{t} D_{jk}(u)|W_k(u-\xi_{jk})|\,du\,dv \\
&+\sum_{k=1}^{n} \int_{t-\xi_{jk}}^{t} H_j(u+\xi_{jk})D_{jk}(u+\xi_{jk})|W_k(u)|\,du,
\end{aligned}
\tag{3.20}
$$

where

$$
H_j(t) = \int_{t-\eta_{jj}}^{t} E_{jj}(u+\eta_{jj})\,du = \int_{t}^{t+\eta_{jj}} E_{jj}(u)\,du. \tag{3.21}
$$

It follows from (3.18)–(3.21) that $V_j(z)(t) \geq 0$ and

$$
\begin{aligned}
\frac{D^+}{Dt}V_j(z)(t) &\leq -E_{jj}(t)|z_j(t)| + \sum_{l=1,l\neq j}^{m} E_{jl}(t+\eta_{jl})|z_l(t)| \\
&+\sum_{l=1}^{m} H_j(t+\eta_{jl})E_{jl}(t+\eta_{jl})|z_l(t)| + \sum_{k=1}^{n} D_{jk}(t+\xi_{jk})|W_k(t)| \\
&+\sum_{k=1}^{n} H_j(t+\xi_{jk})D_{jk}(t+\xi_{jk})|W_k(t)|.
\end{aligned}
\tag{3.22}
$$

On the other hand, for any positive solution $F(t) = (x_1(t),\dots,x_n(t),y_1(t),\dots,y_m(t))$ of (1.3), we have from (1.3) that

$$
\dot{x}_i(t) \leq b_i(t)x_i(t),
$$

which implies

$$
x_i(t) \leq x_i(t-\tau_{ii}) \exp\left\{\int_{t-\tau_{ii}}^{t} b_i(u)\,du\right\}. \tag{3.23}
$$

Similarly, from Lemma 2.6, one can obtain

$$
\dot{y}_j(t) \leq y_j(t)\left[-r_j(t) + \sum_{l=1}^{n} d_{jl}(t)M\right],
$$

which implies

$$
\begin{aligned}
y_j(t) &\leq y_j(t-\eta_{jj}) \exp\left\{\int_{t-\eta_{jj}}^{t} \left[-r_j(u) + \sum_{l=1}^{n} d_{jl}(u)M\right]\,du\right\} \\
&= y_j(t-\eta_{jj}) \exp\left\{r_j^0(t,\eta_{jj})\right\}.
\end{aligned}
\tag{3.24}
$$

From (3.5), (3.6), (3.15), (3.16), (3.23), and (3.24), we can get that

$$\begin{aligned}
A_{ik}(t+\tau_{ik}) &= a_{ik}(t+\tau_{ik})\int_0^1 [x_k(t)]^s[u_k(t)]^{1-s}\,ds\\
&\le a_{ik}(t+\tau_{ik})\int_0^1 [x_k(t-\tau_{kk})]^s\left[u_k(t-\tau_{kk})\exp\left\{b_k^0(t,\tau_{kk})\right\}\right]^{1-s+s}\,ds\\
&= a_{ik}(t+\tau_{ik})\int_0^1 e^{s\tilde{x}_k(t-\tau_{kk})}e^{(1-s)\tilde{u}_k(t-\tau_{kk})}\exp\left\{b_k^0(t,\tau_{kk})\right\}\,ds\\
&= a_{ik}(t+\tau_{ik})\exp\left\{b_k^0(t,\tau_{kk})\right\}\frac{A_{kk}(t)}{a_{kk}(t)}.
\end{aligned}$$

Through similar calculation, we have

$$\begin{aligned}
E_{jl}(t+\eta_{jl}) &\le e_{jl}(t+\eta_{jl})\exp\left\{r_l^0(t,\eta_{ll})\right\}\frac{E_{ll}(t)}{e_{ll}(t)},\\
D_{jk}(t+\xi_{jk}) &\le d_{jk}(t+\xi_{jk})\exp\left\{b_k^0(t,\tau_{kk})\right\}\frac{A_{kk}(t)}{a_{kk}(t)},\\
C_{il}(t+\sigma_{il}) &\le c_{il}(t+\sigma_{il})\exp\left\{r_l^0(t,\eta_{ll})\right\}\frac{E_{ll}(t)}{e_{ll}(t)}.
\end{aligned}$$

Then it follows from (3.12), (3.22) and the definition of matrix $G(t)$ that

$$\begin{aligned}
\frac{D^+}{Dt}V_i(W)(t) \le& -\frac{A_{ii}(t)}{a_{ii}(t)}\left[a_{ii}(t)-B_i(t+\tau_{ii})a_{ii}(t+\tau_{ii})\exp\left\{b_i^0(t,\tau_{ii})\right\}\right]|W_i(t)|\\
&+\sum_{k=1,k\ne i}^{n}\left[1+B_i(t+\tau_{ik})\right]a_{ik}(t+\tau_{ik})\exp\left\{b_k^0(t,\tau_{kk})\right\}\frac{A_{kk}(t)}{a_{kk}(t)}|W_k(t)|\\
&+\sum_{l=1}^{m}\left[1+B_i(t+\sigma_{il})\right]c_{il}(t+\sigma_{jl})\exp\left\{r_l^0(t,\eta_{ll})\right\}\frac{E_{ll}(t)}{e_{ll}(t)}|z_l(t)| \qquad (3.25)\\
=& -\sum_{k=1}^{n}f_{ik}(t)\frac{A_{kk}(t)}{a_{kk}(t)}|W_k(t)|-\sum_{l=1}^{m}g_{il}(t)\frac{E_{ll}(t)}{e_{ll}(t)}|z_l(t)|,\\
\frac{D^+}{Dt}V_j(z)(t) \le& -\frac{E_{jj}(t)}{e_{jj}(t)}\left[e_{jj}(t)-H_j(t+\eta_{jj})e_{jj}(t+\eta_{jj})\exp\left\{r_j^0(t,\eta_{jj})\right\}\right]|z_j(t)|\\
&+\sum_{l=1,l\ne j}^{m}\left[1+H_j(t+\eta_{jl})\right]e_{jl}(t+\eta_{jl})\exp\left\{r_l^0(t,\eta_{ll})\right\}\frac{E_{ll}(t)}{e_{ll}(t)}|z_l(t)|\\
&+\sum_{k=1}^{n}\left[1+H_j(t+\xi_{jk})\right]d_{jk}(t+\xi_{jk})\exp\left\{b_k^0(t,\tau_{kk})\right\}\frac{A_{kk}(t)}{a_{kk}(t)}|W_k(t)| \qquad (3.26)\\
=& -\sum_{l=1}^{m}w_{jl}(t)\frac{E_{ll}(t)}{e_{ll}(t)}|z_l(t)|-\sum_{k=1}^{n}h_{jk}(t)\frac{A_{kk}(t)}{a_{kk}(t)}|W_k(t)|.
\end{aligned}$$

Since

$$(G(t))_{(n+m)\times(n+m)} = \begin{pmatrix} (f_{ik}(t))_{n\times n} & (g_{il}(t))_{n\times m}\\ (h_{jk}(t))_{m\times n} & (w_{jl}(t))_{m\times m}\end{pmatrix}$$

is a generalized uniform M-matrix, there exist positive vector $\alpha=(s_1,\dots,s_n,\theta_1,\dots,\theta_m)^\top$ and a nonnegative function vector $\beta(t)=(\lambda_1(t),\dots,\lambda_n(t),\mu_1(t),\dots,\mu_m(t))^\top\in R^{n+m}$ such that

$$\begin{pmatrix} (f_{ik}(t))_{n\times n} & (g_{il}(t))_{n\times m}\\ (h_{jk}(t))_{m\times n} & (w_{jl}(t))_{m\times m}\end{pmatrix}^\top\begin{pmatrix}(s_i)_{n\times 1}\\(\theta_j)_{m\times 1}\end{pmatrix}\ge\begin{pmatrix}(\lambda_i(t))_{n\times 1}\\(\mu_j(t))_{m\times 1}\end{pmatrix}$$

or

$$\sum_{k=1}^{n} s_k f_{ki}(t) + \sum_{l=1}^{m} \theta_l h_{li}(t) \geq \lambda_i(t), \tag{3.27}$$

$$\sum_{l=1}^{m} \theta_l w_{lj}(t) + \sum_{k=1}^{n} s_k g_{lj}(t) \geq \mu_j(t). \tag{3.28}$$

Now define

$$V(t) = \sum_{i=1}^{n} s_i V_i(W)(t) + \sum_{j=1}^{m} \theta_j V_j(z)(t). \tag{3.29}$$

Calculating the upper right derivative $\left(\frac{D^+}{Dt}\right) V$ of V, and from (3.3), (3.5), (3.16), (3.25), and (3.26), we have

$$\begin{aligned}
\frac{D^+}{Dt} V(t) &\leq -\sum_{i=1}^{n} s_i \left[\sum_{k=1}^{n} f_{ik}(t) \frac{A_{kk}(t)}{a_{kk}(t)} |W_k(t)| + \sum_{l=1}^{m} g_{il}(t) \frac{E_{ll}(t)}{e_{ll}(t)} |z_l(t)|\right] \\
&\quad + \sum_{j=1}^{m} \theta_j \left[\sum_{l=1}^{m} w_{jl}(t) \frac{E_{ll}(t)}{e_{ll}(t)} |z_l(t)| + \sum_{k=1}^{n} h_{jk}(t) \frac{A_{kk}(t)}{a_{kk}(t)} |W_k(t)|\right] \\
&= -\sum_{i=1}^{n} \left[\sum_{k=1}^{n} s_k f_{ki}(t) + \sum_{l=1}^{m} \theta_l h_{li}(t)\right] \frac{A_{ii}(t)}{a_{ii}(t)} |W_i(t)| \\
&\quad - \sum_{j=1}^{m} \left[\sum_{l=1}^{m} \theta_l w_{lj}(t) + \sum_{k=1}^{n} s_k g_{lj}(t)\right] \frac{E_{jj}(t)}{e_{jj}(t)} |z_j(t)| \\
&\leq -\sum_{i=1}^{n} \lambda_i(t) \frac{A_{ii}(t)}{a_{ii}(t)} |W_i(t)| - \sum_{j=1}^{m} \mu_j(t) \frac{E_{jj}(t)}{e_{jj}(t)} |z_j(t)| \\
&\leq -\gamma(t) \left[\sum_{i=1}^{n} |x_i(t) - u_i(t)| + \sum_{j=1}^{m} |y_j(t) - v_j(t)|\right], \qquad \text{for } t \geq 0,
\end{aligned} \tag{3.30}$$

where $\gamma(t) = \min_{i,j}\{\lambda_i(t), \mu_j(t)\}$. Integrating (3.30) on $[0, t]$, we can get that

$$\int_0^t \gamma(s) \left[\sum_{i=1}^{n} |x_i(s) - u_i(s)| + \sum_{j=1}^{m} |y_j(s) - v_j(s)|\right] ds < V(0) - V(t), \qquad \text{for all } t \geq 0.$$

Hence, we have

$$\int_0^t \gamma(s) \left[\sum_{i=1}^{n} |x_i(s) - u_i(s)| + \sum_{j=1}^{m} |y_j(s) - v_j(s)|\right] ds < \infty, \qquad \text{for all } t \geq 0. \tag{3.31}$$

By the uniform boundedness of the positive solutions of (1.3), we obtain from system (1.3) that $|x_i(t) - u_i(t)|$, $|y_j(t) - v_j(t)|$ are bounded and uniformly continuous on $[0, \infty)$. Hence, by the periodicity of $\gamma(t)$, it follows from (3.31) that

$$\lim_{t \to \infty} |x_i(t) - u_i(t)| \to 0, \qquad \lim_{t \to \infty} |y_j(t) - v_j(t)| \to 0,$$

which implies the global attractivity of the unique T-periodic solution of (1.3). This completes the proof of the main theorem.

COROLLARY 3.1. *In addition to* (H_1)–(H_6), *if system (1.3),(2.1) also satisfies*

$$p_i \bar{B}_{ii}(t,\tau_{ii}) a_{ii}(t+\tau_{ii}) \exp\left\{b_i^0(t,\tau_{ii})\right\} < a_{ii}(t),$$
$$q_j \bar{H}_{jj}(t,\eta_{jj}) e_{jj}(t+\eta_{jj}) \exp\left\{r_j^0(t,\eta_{jj})\right\} < e_{jj}(t),$$

and

$$(\bar{G}(t))_{(n+m)\times(n+m)} = \begin{pmatrix} \left(\bar{f}_{ik}(t)\right)_{n\times n} & \left(\bar{g}_{il}(t)\right)_{n\times m} \\ \left(\bar{h}_{jk}(t)\right)_{m\times n} & \left(\bar{w}_{jl}(t)\right)_{m\times m} \end{pmatrix}$$

is an eventually uniform M*-matrix, where* $(\bar{G}(t))_{(n+m)\times(n+m)}$ *is defined by*

$$\bar{B}_{ii}(t,\tau_{ik}) = \int_t^{t+\tau_{ii}} a_{ii}(u+\tau_{ik})\,du, \qquad \bar{B}_{ii}(t,\sigma_{il}) = \int_t^{t+\tau_{ii}} a_{ii}(u+\sigma_{il})\,du,$$
$$\bar{H}_{jj}(t,\eta_{jl}) = \int_t^{t+\eta_{jj}} e_{jl}(u+\eta_{jl})\,du, \qquad \bar{H}_{jj}(t,\xi_{jk}) = \int_t^{t+\eta_{jj}} e_{jl}(u+\xi_{jk})\,du,$$
$$\bar{f}_{ik}(t) = \begin{cases} a_{ii}(t) - p_i \bar{B}_{ii}(t,\tau_{ii}) a_{ii}(t+\tau_{ii}) \exp\{b_i^0(t,\tau_{ii})\}, & i = k, \\ -\left[1 + p_i \bar{B}_{ii}(t,\tau_{ik})\right] a_{ik}(t+\tau_{ik}) \exp\{b_k^0(t,\tau_{kk})\}, & i \neq k, \end{cases}$$
$$\bar{h}_{jk}(t) = -\left[1 + q_j \bar{H}_{jj}(t,\xi_{jk})\right] d_{jk}(t+\xi_{jk}) \exp\{b_k^0(t,\tau_{kk})\},$$
$$\bar{g}_{il}(t) = -\left[1 + p_i \bar{B}_{ii}(t,\sigma_{il})\right] c_{il}(t+\sigma_{il}) \exp\{r_l^0(t,\eta_{ll})\},$$
$$\bar{w}_{jl}(t) = \begin{cases} e_{jj}(t) - q_j \bar{H}_{jj}(t,\eta_{jj}) e_{jj}(t+\eta_{jj}) \exp\{r_j^0(t,\eta_{jj})\}, & j = k, \\ -\left[1 + q_j \bar{H}_{jj}(t,\eta_{jl})\right] e_{jl}(t+\eta_{jl}) \exp\{r_l^0(t,\eta_{ll})\}, & j \neq k, \end{cases}$$

where $b_k^0(t,\tau_{kk})$ *and* $r_k^0(t,\eta_{kk})$ *defined as in Theorem 3.1, then system (1.3) has a unique globally attractive positive* T*-periodic solution.*

PROOF. Proof is similar to that of Theorem 3.1. From Lemma 2.6, there exists a $T_1 \geq 0$ such that, for $t \geq T_1$,

$$B_i(t+\tau_{ik}) = \int_t^{t+\tau_{ii}} A_{ii}(u+\tau_{ik})\,du \leq p_i \int_t^{t+\tau_{ii}} a_{ii}(u+\tau_{ik})\,du;$$
$$B_i(t+\sigma_{il}) = \int_t^{t+\tau_{ii}} A_{ii}(u+\sigma_{il})\,du \leq p_i \int_t^{t+\tau_{ii}} a_{ii}(u+\sigma_{il})\,du;$$
$$H_j(t+\eta_{jl}) = \int_t^{t+\eta_{jj}} E_{jj}(u+\eta_{jl})\,du \leq q_j \int_t^{t+\eta_{jj}} e_{jl}(u+\eta_{jl})\,du;$$
$$H_j(t+\xi_{jk}) = \int_t^{t+\eta_{jj}} E_{jj}(u+\xi_{jk})\,du \leq q_j \int_t^{t+\eta_{jj}} e_{jl}(u+\xi_{jk})\,du.$$

Since $(\bar{G}(t))_{(n+m)\times(n+m)}$ is an eventually uniform M-matrix, there exist positive vectors $\alpha = (s_1,\dots,s_n,\theta_1,\dots,\theta_m)^\top$, $\beta = (\lambda_1,\dots,\lambda_n,\mu_1,\dots,\mu_m)^\top \in R^{n+m}$ and $T_0 \geq T_1$ such that $\bar{G}^\top(t)\alpha \geq \beta$ for $t \geq T_0$, i.e.,

$$\begin{pmatrix} \left(\bar{f}_{ik}(t)\right)_{n\times n} & \left(\bar{g}_{il}(t)\right)_{n\times m} \\ \left(\bar{h}_{jk}(t)\right)_{m\times n} & \left(\bar{w}_{jl}(t)\right)_{m\times m} \end{pmatrix}^\top \begin{pmatrix} (s_i)_{n\times 1} \\ (\theta_j)_{m\times 1} \end{pmatrix} \geq \begin{pmatrix} (\lambda_i)_{n\times 1} \\ (\mu_j)_{m\times 1} \end{pmatrix}.$$

Calculating the upper right derivative $\left(\frac{D^+}{Dt}\right) V(t)$ of $V(t)$ (defined in (3.29)), and from (3.25), (3.26), we have

$$\frac{D^+}{Dt} V(t) \leq -\sum_{i=1}^n \lambda_i \frac{A_{ii}(t)}{a_{ii}(t)} |W_i(t)| - \sum_{j=1}^m \mu_j \frac{E_{jj}(t)}{e_{jj}(t)} |z_j(t)|, \qquad \text{for } t \geq T_0. \tag{3.32}$$

Let $\gamma = \min_{i,j}\{\lambda_i, \mu_j\}$, then (3.32) leads to

$$\frac{D^+}{Dt}V(t) \le -\gamma \left[\sum_{i=1}^{n} |x_i(t) - u_i(t)| + \sum_{j=1}^{m} |y_j(t) - v_j(t)|\right], \qquad \text{for } t \ge T_0. \tag{3.33}$$

Integrating (3.33) on $[T_0, \infty)$, it follows that

$$V(t) + \frac{\gamma}{x^*}\int_{T_0}^{\infty} \left[\sum_{i=1}^{n} |x_i(s) - u_i(s)| + \sum_{j=1}^{m} |y_j(s) - v_j(s)|\right] ds < V(T_0) < +\infty.$$

Therefore,

$$\limsup_{t\to\infty} \int_{T_0}^{\infty} \left[\sum_{i=1}^{n} |x_i(s) - u_i(s)| + \sum_{j=1}^{m} |y_j(s) - v_j(s)|\right] ds < \frac{x^* V(T_0)}{\gamma} < +\infty.$$

Hence,

$$\lim_{t\to\infty} |x_i(t) - u_i(t)| \to 0, \qquad \lim_{t\to\infty} |y_j(t) - v_j(t)| \to 0.$$

This completes the proof.

4. BOUNDEDNESS AND EXISTENCE OF PERIODIC SOLUTIONS

THEOREM 4.1. *Suppose* (H_1)–(H_6) *hold, then all the positive solutions of system (1.3),(2.1) are uniformly bounded and uniformly ultimately bounded both above and below by positive constant vectors.*

PROOF.

STEP 1. We shall prove that the solutions of the initial value problem (1.3),(2.1) are uniformly ultimately bounded above and below by positive constant vectors at $t = 0$.

From the first equation of (1.3), we have

$$\dot{x}_i(t) \le x_i(t)\left[b_i(t) - a_{ii}(t)x_i(t - \tau_{ii})\right].$$

It follows that

$$\dot{x}_i(t) \le b_i^{\mu} x_i(t). \tag{4.1}$$

An integration of (4.1) over $[t - \tau_{ii}, t]$, one obtains

$$x_i(t - \tau_{ii}) \ge x_i(t)\exp\left\{-b_i^{\mu}\tau_{ii}\right\}, \qquad t \ge \tau_{ii}. \tag{4.2}$$

From the first equation of (1.3) and (4.2), we have

$$\dot{x}_i(t) \le x_i(t)\left[b_i(t) - a_{ii}(t)\exp\left\{-b_i^{\mu}\tau_{ii}\right\}x_i(t)\right]. \tag{4.3}$$

From (H_1), using the comparison theorem on (4.3), one obtains

$$x_i(t) \le x_i^*(t), \qquad t \ge \tau_{ii},$$

where $x_i(t)$ is a solution of (1.3) which satisfies $0 < \max\{x_i(\theta + \tau_{ii}), \theta \in [-\tau, 0]\} \le x_i^*(\tau_{ii})$.

We note that (H_1) implies $b_i^{\mu} \ge 0$, then it is not difficult to obtain that

$$\lim_{t\to\infty} x_i^*(t) \le p_i, \qquad \text{for all } t \in R.$$

Thus, there exists a $t_i^{\mathrm{I}} \geq \tau_{ii} > 0$ such that

$$x_i(t) \leq p_i, \qquad \text{for all } t \geq t_i^{\mathrm{I}}. \tag{4.4}$$

We choose $P = \max_i p_i$, then $x_i(t) \leq P$ for all $t \geq \max_{1\leq i\leq n}\{t_i^{\mathrm{I}}\}$.

Let $t_j^{\mathrm{II}} = \max_{1\leq i\leq n}\{t_i^{\mathrm{I}}\} + \eta_{jj}$, if $t \geq \max_{1\leq i\leq n}\{t_i^{\mathrm{I}}\}$, from the second equation of (1.3) and (4.4), it follows that

$$\dot{y}_j(t) \leq y_j(t)\left[-r_j^{\iota} + \sum_{k=1}^{n} d_{jk}^{\mu} p_k\right]. \tag{4.5}$$

An integration of (4.5) over $[t - \eta_{jj}, t]$ leads to

$$y_j(t-\eta_{jj}) \geq y_j(t)\exp\left\{-\left(-r_j^{\iota} + \sum_{k=1}^{n} d_{jk}^{\mu} p_k\right)\eta_{jj}\right\}, \qquad t \geq t_j^{\mathrm{II}}. \tag{4.6}$$

From the second equation of (1.3) and (4.6), one obtains

$$\begin{aligned}\dot{y}_j(t) \leq y_j(t) \Bigg[&-r_j(t) + \sum_{k=1}^{n} d_{jk}(t) x_k^*(t-\xi_{jk}) \\ &- e_{jj}(t)\exp\left\{-\left(-r_j^{\iota} + \sum_{k=1}^{n} d_{jk}^{\mu} p_k\right)\eta_{jj}\right\} y_j(t)\Bigg].\end{aligned} \tag{4.7}$$

From (H_2), using the comparison theorem on (4.7), one obtains

$$y_j(t) \leq y_j^*(t), \qquad t \geq t_j^{\mathrm{II}},$$

where $y_j(t)$ is a solution of (1.3) which satisfies $0 < \max\{y_j(\theta + t_j^{\mathrm{II}}), \theta \in [-\tau, 0]\} \leq y_j^*(t_j^{\mathrm{II}})$.

We note that (H_2) implies $-r_j^{\iota} + \sum_{k=1}^{n} d_{jk}^{\mu} p_k > 0$, then it is not difficult to obtain that

$$\lim_{t\to\infty} y_j^*(t) \leq q_j, \qquad \text{for all } t \in R.$$

Thus, there exists a $t_j^{\mathrm{III}} \geq t_j^{\mathrm{II}} > 0$ such that

$$y_j(t) \leq q_j, \qquad \text{for all } t \geq t_j^{\mathrm{III}}. \tag{4.8}$$

We choose $Q = \max_j q_j$, then $y_j(t) \leq Q$ for all $t \geq \max_{1\leq j\leq m}\{t_j^{\mathrm{III}}\}$.

Let $t^{\mathrm{IV}} = \max_{1\leq j\leq m}\{t_j^{\mathrm{III}}\}$, if $t \geq t^{\mathrm{IV}}$, from the first equation of (1.3) and the above discussion, one obtains

$$\dot{x}_i(t) \geq x_i(t)\left[b_i^{\iota} - \sum_{k=1}^{n} a_{ik}^{\mu} p_k - \sum_{l=1}^{m} c_{il}^{\mu} q_l\right]. \tag{4.9}$$

An integration of (4.9) over $[t - \tau_{ii}, t]$ leads to

$$x_i(t-\tau_{ii}) \leq \exp\left\{-\left[b_i^{\iota} - \sum_{k=1}^{n} a_{ik}^{\mu} p_k - \sum_{l=1}^{m} c_{il}^{\mu} q_l\right]\tau_{ii}\right\} x_i(t), \qquad t \geq t^{\mathrm{IV}} + \tau_{ii}. \tag{4.10}$$

From the first equation of (1.3) and (4.10), one can get

$$\dot{x}_i(t) \geq x_i(t)\left[B_i(t) - A_i(t)x_i(t)\right]. \tag{4.11}$$

From (H_3), we can obtain that there exist positive constants ε_i, γ_i such that

$$\int_t^{t+T} \left[B_i(t) - A_i(t)\varepsilon_i\right] dt > \gamma_i, \qquad \text{for all } t \geq 0. \tag{4.12}$$

Considering the following auxiliary equation:

$$\dot{u}_i(t) = u_i(t)[B_i(t) - A_i(t)u_i(t)], \tag{4.13}$$

using the comparison theorem on (4.11), we have

$$x_i(t) \geq u_i^0(t), \qquad \text{for all } t \geq t^{\mathrm{IV}} + \tau_{ii}, \tag{4.14}$$

where $u_i^0(t)$ is the solution of equation (4.13) with initial condition $u_i^0(t_0) \leq \min\{x_i(t_0+\theta), \theta \in [-\tau, 0]\}$, $t_0 = t^{\mathrm{IV}} + \tau_{ii}$.

If $u_i^0(t_0) \leq \varepsilon_i$ for all $t \geq t_0$, then $u_i^0(t_0)$ defined on $[t_0, +\infty)$. From equation (4.13), it follows that

$$u_i^0(t) = u_i^0(t_0) \exp \int_{t_0}^{t} [B_i(s) - A_i(s)u_i(s)]\, ds \geq u_i^0(t_0) \exp \int_{t_0}^{t} [B_i(s) - A_i(s)\varepsilon_i]\, ds, \tag{4.15}$$

for all $t \geq t_0$. Choose $t = t_0 + pT$, $p = 1, 2, \dots$, then (4.12) and (4.15) imply that

$$u_i^0(t_0 + pT) \geq u_i^0(t) \exp(p\gamma_i), \qquad \text{for } p = 1, 2, \dots.$$

Constantly, $\lim_{p\to\infty} u_i^0(t_0 + pT) = \infty$, a contradiction. Hence, there is a $t_i^o > t_0$ such that $u_i^0(t_i^o) > \varepsilon_i$. We now prove that

$$u_i^0(t) \geq \varepsilon_i \exp(-\varpi_i(\varepsilon_i)T), \tag{4.16}$$

for all $t \geq t_i^o$, where

$$\varpi_i(\varepsilon_i) = \max\{|B_i(t)| + A_i(t)\varepsilon_i : t \in [0, \infty)\}.$$

From the periodicity of the coefficients of (1.3) and the above analysis, it is easy to obtain that $0 < \varpi_i(\varepsilon_i) < \infty$. It is obvious that $\varpi_i(\varepsilon_i)$ is independent of any positive solution of system (1.3). Suppose that (4.16) is not true, then there are t_1 and t_2, $t_2 > t_1 > t_i^o$, such that $u_i^0(t_2) < \varepsilon_i \exp(-\varpi_i(\varepsilon_i)T)$, $u_i^0(t_1) = \varepsilon_i$ and $u_i^0(t) < \varepsilon_i$ for $t \in (t_1, t_2]$. Choose an integer $p \geq 0$ such that $t_2 \in (t_1 + pT, t_1 + (p+1)T]$, then by (4.12) and (4.13), it follows that

$$\begin{aligned}
\varepsilon_i \exp(-\varpi_i(\varepsilon_i)T) > u_i^0(t_2) &\geq \varepsilon_i \exp \int_{t_1}^{t_2} [B_i(t) - A_i(t)\varepsilon_i]\, dt \\
&= \varepsilon_i \exp \left[\int_{t_1}^{t_1+pT} + \int_{t_1+pT}^{t_2} [B_i(t) - A_i(t)\varepsilon_i] \right] dt \\
&\geq \varepsilon_i \exp \int_{t_1+pT}^{t_2} [B_i(t) - A_i(t)\varepsilon_i]\, dt \geq \varepsilon_i \exp(-\varpi_i(\varepsilon_i)T),
\end{aligned}$$

which is a contradiction.

From (4.14) and (4.16), we obtain

$$x_i(t) \geq \varepsilon_i \exp(-\varpi_i(\varepsilon_i)T), \qquad \text{for all } t \geq t_i^o.$$

Finally, we choose $\alpha = \min_{1\leq i\leq n}\{\varepsilon_i \exp(-\varpi_i(\varepsilon_i)T)\}$ and $t^{\mathrm{V}} = \max_{1\leq i\leq n}\{t_i^o\}$, then $x_i(t) \geq \alpha$ for all $t \geq t^{\mathrm{V}}$.

Now consider $y_j(t)$ for $t \geq t^{\mathrm{V}}$, then from the first equation of (1.3) and the above analysis, one obtains

$$\dot{y}_j(t) \geq y_j(t) \left[-r_j^{\mu} + \sum_{k=1}^{n} d_{jk}^{\iota}\alpha - \sum_{l=1}^{m} e_{jl}^{\mu} q_l \right]. \tag{4.17}$$

An integration of (4.17) over $[t-\eta_{jj}, t]$ leads to

$$y_j(t-\eta_{jj}) \le \exp\left\{-\left[-r_j^\mu + \sum_{k=1}^{n} d_{jk}^\iota \alpha - \sum_{l=1}^{m} e_{jl}^\mu q_l\right]\eta_{jj}\right\} y_j(t), \qquad t \ge t^{\mathrm{V}} + \eta_{jj}. \tag{4.18}$$

From the second equation of (1.3) and (4.17), one can get

$$\begin{aligned}\dot{y}_j(t) \ge y_j(t) &\left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t) u_k^o(t-\xi_{jk}) - \sum_{l=1, l\ne j}^{m} e_{jl}(t) y_l^*(t-\eta_{jl})\right. \\ &\left. - e_{jj}(t)\exp\left\{-\left[-r_j^\mu + \sum_{k=1}^{n} d_{jk}^\iota \alpha - \sum_{l=1}^{m} e_{jl}^\mu q_l\right]\eta_{jj}\right\} y_j(t)\right].\end{aligned} \tag{4.19}$$

From (H_4), through the similar proof as $x_i(t)$, there exist a constant $\beta > 0$ and $t^{\mathrm{VI}} \ge t^{\mathrm{V}}$ such that $y_j(t) \ge \beta$, for all $t \ge t^{\mathrm{VI}}$.

STEP 2. Now we prove that the solutions of the initial value problem (1.3),(2.1) are uniformly bounded by positive constant vectors at $t = 0$.

First, we consider $x_i(t)$ for all $t \ge 0$. If $t \in [0, \tau_{ii}]$, from (4.1), it follows that

$$x_i(t) \le x_i(0)\exp\left\{\int_0^t b_i^\mu \, ds\right\} \le x_i(0)\exp\left\{b_i^\mu \tau_{ii}\right\} < \delta_i^\mu \exp\left\{b_i^\mu \tau_{ii}\right\},$$

then from Step 1, it follows that $x_i(t) \le \max\{\delta_i^\mu \exp\{b_i^\mu \tau_{ii}\}, \max_{t\in[0,T]} x_i^*(t)\}$, for all $t \ge 0$. Therefore, $x_i(t) \le M = \max_i\{\delta_i^\mu \exp\{b_i^\mu \tau_{ii}\}, \max_{t\in[0,T]} x_i^*(t)\}$, for all $t \ge 0$.

Similar discussion as in Step 1, one obtains

$$\begin{aligned}\dot{y}_j(t) \le y_j(t) &\left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t) x_k^*(t-\xi_{jk})\right. \\ &\left. - e_{jj}(t)\exp\left\{-\left(-r_j^\iota + \sum_{k=1}^{n} d_{jk}^\mu M\right)\eta_{jj}\right\} y_j(t)\right].\end{aligned} \tag{4.20}$$

From (H_2), using the comparison theorem on (4.20), one obtains

$$y_j(t) \le \hat{y}_j^*(t), \qquad t \ge \eta_{jj},$$

where $y_j(t)$ is a solution of (1.3) which satisfies $0 < \max\{y_j(\theta+\eta_{jj}), \theta \in [-\tau, 0]\} \le \hat{y}_j^*(\eta_{jj})$.

On the other hand, if $t \in [0, \eta_{jj}]$, integrating

$$\dot{y}_j(t) \le y_j(t)\left[-r_j^\iota + \sum_{k=1}^{n} d_{jk}^\mu M\right]$$

over $[0, t]$, it follows that

$$y_j(t) \le y_j(0)\exp\left\{\int_0^t \left[-r_j^\iota + \sum_{k=1}^{n} d_{jk}^\mu M\right] ds\right\} < \rho_j^\mu \exp\left\{\left[-r_j^\iota + \sum_{k=1}^{n} d_{jk}^\mu M\right]\eta_{jj}\right\},$$

then it follows that

$$y_j(t) \le \max\left\{\rho_j^\mu \exp\left\{\left[-r_j^\iota + \sum_{k=1}^{n} d_{jk}^\mu M\right]\eta_{jj}\right\}, \max_{t\in[0,T]} \hat{y}_j^*(t)\right\},$$

for all $t \geq 0$. Therefore,

$$y_j(t) \leq M^* = \max_j \left\{ \rho_j^\mu \exp\left\{ \left[-r_j^\iota + \sum_{k=1}^n d_{jk}^\mu M \right] \eta_{jj} \right\}, \max_{t\in[0,T]} \hat{y}_j^*(t) \right\},$$

for all $t \geq 0$.

From the above discussion, (4.9) can be written as

$$\dot{x}_i(t) \geq x_i(t) \left[b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^* \right]. \tag{4.21}$$

If $t \in [0, \tau_{ii}]$, an integration of (4.21) over $[0, t]$, one can see that

$$x_i(t) \geq x_i(0) \exp\left\{ \left[b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^* \right] t \right\}. \tag{4.22}$$

From (4.22), it is easy to see that if $b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^* \geq 0$, then $x_i(t_{L_i}^\iota) \geq x_i(0) > \delta_i^\iota$, and if $b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^* \leq 0$, then

$$x_i(t) \geq \delta_i^\iota \exp\left\{ \tau_{ii} \left[b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^* \right] \right\}.$$

Obviously,

$$x_i(t) \geq \delta_i^\iota \exp\left\{ \tau_{ii} \left[b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^* \right] \right\}$$

in both cases.

On the other hand, from (4.21) and similar discussion as in Step 1, one can get

$$\begin{aligned} \dot{x}_i(t) \geq x_i(t) \Bigg[& b_i(t) - \sum_{k=1, k\neq i}^n a_{ik}(t) x_k^*(t - \tau_{ik}) - \sum_{l=1}^m c_{il}(t) \hat{y}_l^*(t - \sigma_{il}) \\ & - a_{ii}(t) \exp\left\{ - \left[b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^* \right] \tau_{ii} \right\} x_i(t) \Bigg]. \end{aligned} \tag{4.23}$$

From (H_5), using the comparison theorem on (4.23), one obtains

$$x_i(t) \leq \hat{u}_j^o(t), \qquad t \geq \tau_{ii},$$

where $x_i(t)$ is a solution of (1.3) which satisfies $\min\{x_i(\theta + \tau_{ii}),\ \theta \in [-\tau, 0]\} \geq \hat{u}_i^o(\tau_{ii})$.

Let

$$\bar{\alpha} = \min_i \left\{ \delta_i^\iota \exp\left\{ \tau_{ii} \left[b_i^\iota - \sum_{k=1}^n a_{ik}^\mu M - \sum_{l=1}^m c_{il}^\mu M^* \right] \right\}, \min_{t\in[0,T]} \hat{u}_i^o(\tau_{ii}) \right\},$$

then $x_i(t) \geq \bar{\alpha}$ for all $t \geq 0$.

From (H_6), through the similar proof as $x_i(t)$, one can obtain that there exists a $\bar{\beta} > 0$ such that $y_j(t) \geq \bar{\beta}$ for all $t \geq 0$. This completes the proof of Theorem 4.1.

Following we will establish the existence of positive periodic solutions of system (1.3) by employing Theorem 4.2.2 of [23].

Theorem 4.2. *If* (H_1)–(H_6) *hold, then system (1.3) has at least one positive* T*-periodic solution.*

Proof. Denote

$$
\begin{aligned}
f_i(t,F_t) &= b_i(t) - \sum_{k=1}^{n} a_{ik}(t)x_k(t-\tau_{ik}) - \sum_{l=1}^{m} c_{il}(t)y_l(t-\sigma_{il}), \qquad i=1,2,\dots,n,\\
g_j(t,F_t) &= -r_j(t) + \sum_{k=1}^{n} d_{jk}(t)x_k(t-\xi_{jk}) - \sum_{l=1}^{m} e_{jl}(t)y_l(t-\eta_{jl}), \qquad j=1,2,\dots,m.
\end{aligned}
$$

Then (1.3) can be written in the form

$$
\begin{aligned}
\dot{x}_i(t) &= x_i(t)f_i(t,F_t),\\
\dot{y}_j(t) &= y_j(t)g_j(t,F_t).
\end{aligned} \tag{4.24}
$$

To show the existence of a positive periodic solution of (1.3), we consider the solutions of (1.3) with initial functions $\Phi(s) > 0$ for $s \in [-\tau, 0]$. Let $F(t) = F(t,\Phi)$ be the solution of (4.24) corresponding to such a $\Phi \in \mathbf{C}^+(0)$ and define $Z(t) = (u_1(t),\dots,u_n(t),v_1(t),\dots,v_m(t))$ with $u_i(t) = \ln x_i(t), v_j(t) = \ln y_j(t)$. Then $Z(t)$ satisfies

$$
\begin{aligned}
\dot{u}_i(t) &= f_i(t,e^{Z_t}) \triangleq \tilde{f}_i(t,Z_t),\\
\dot{v}_j(t) &= g_j(t,e^{Z_t}) \triangleq \tilde{g}_j(t,Z_t),
\end{aligned} \tag{4.25}
$$

for $t \ge 0$ and $Z(s) = \Psi(s)$, for $s \in [-\tau, 0]$ with $\Psi(s) = (\xi_1(s),\dots,\xi_n(s),\eta_1(s),\dots,\eta_m(s))$ and $\xi_i(s) = \ln\phi_i(s), \eta_j(s) = \ln\psi_j(s)$. Obviously, $\Psi \in \mathbf{C}(0)$. Then it follows from Theorem 4.1 and $Z(t) = \ln F(t)$ that the solutions of (4.25) are uniform bounded and uniform ultimate bounded (in the sense of Burton's definitions, see [23, p. 248]). And also, by the periodicity of $b_i(t), r_j(t), a_{ik}(t), c_{il}(t)$ and $d_{jk}(t), e_{jl}(t)$, if $Z(t)$ is a periodic solution of (4.25) with period $T > 0$, then $Z(t+T)$ is also a T-periodic solution of (4.25). It then follows from Theorem 4.2.2 of [23] that (4.25) has a T-periodic solution, say $\tilde{Z}(t)$. Let $\tilde{F}(t) = \exp(\tilde{Z})$. Then obviously $\tilde{F}(t)$ is a positive T-periodic solution of (1.3). This completes the proof.

5. APPLICATIONS

To illustrate the generality of our results, we shall apply the results obtained in Sections 3 and 4 to particular predator-and-prey system and competition systems, which have been studied extensively in the literature. The following four applications will show that our easily verifiable sufficient conditions are more general and weaker than those available in the literature, thus improve and generalize some well-known results.

Application 5.1. When $\tau_{ii} \equiv 0$, $\eta_{jj} \equiv 0$, i.e., system (1.3) reduces to system (1.2). In this case, Conditions (H_1)–(H_6) degenerate to (H_1)–(H_4). Now applying our main results Theorem 3.1 to system (1.2), one can reach the following.

Theorem 5.1. *Suppose* (H_1)–(H_4) *hold, and*

(H_7) *there exist strictly positive constants* s_i, θ_j *and positive* T*-periodic functions* $\lambda_i(t)$ *and* $\mu_j(t)$ *such that*

$$
\begin{aligned}
s_i a_{ii}(t) &\ge \sum_{k=1,k\ne i}^{n} s_k a_{ki}(t+\tau_{ik}) + \sum_{l=1}^{m} \theta_l d_{li}(t+\xi_{li}) + \lambda_i(t),\\
\theta_j e_{jj}(t) &\ge \sum_{k=1}^{n} s_k c_{kj}(t+\sigma_{kj}) + \sum_{l=1,l\ne j}^{m} \theta_l e_{lj}(t+\eta_{lj}) + \mu_j(t).
\end{aligned}
$$

Then system (1.2) has a unique T-periodic and strictly positive solution which is globally attractive.

REMARK 5.1. Let $\lambda_i(t) \equiv \mu_j(t) \equiv \gamma(t)$, one can easily reach the results in [22].

APPLICATION 5.2. Consider the predator-prey system without any delay, i.e., system (1.1). In this special case, Conditions (H_1)–(H_6) reduce to (H_1) and

$$(\bar{H}_2) \qquad \int_0^T \left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t)\tilde{x}_k^*(t) \right] dt > 0,$$

$$(\bar{H}_3) \qquad \int_0^T \left[b_i(t) - \sum_{k=1,k\neq i}^{n} a_{ik}(t)\tilde{x}_k^*(t) - \sum_{l=1}^{m} c_{il}(t)\tilde{y}_l^*(t) \right] dt > 0,$$

$$(\bar{H}_4) \qquad \int_0^T \left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t)\tilde{u}_k^o(t) - \sum_{l=1,l\neq j}^{m} e_{jl}(t)\tilde{y}_l^*(t) \right] dt > 0.$$

THEOREM 5.2. *Suppose* (H_1), ($\bar{H}_2$)–($\bar{H}_4$) *hold, and*

(H_8) *there exist strictly positive constants s_i, θ_j and positive T-periodic functions $\lambda_i(t)$ and $\mu_j(t)$ such that*

$$s_i a_{ii}(t) \geq \sum_{k=1,k\neq i}^{n} s_k a_{ki}(t) + \sum_{l=1}^{m} \theta_l d_{li}(t) + \lambda_i(t),$$
$$\theta_j e_{jj}(t) \geq \sum_{k=1}^{n} s_k c_{kj}(t) + \sum_{l=1,l\neq j}^{m} \theta_l e_{lj}(t) + \mu_j(t).$$

Then system (1.1) has a unique T-periodic and strictly positive solution which is globally attractive.

REMARK 5.2. Yang and Xu [19] have studied system (1.1) with $\inf_{t\in R} b_i(t) > 0$, then Zhao and Chen [20] considered system (1.1) with $\int_0^T b_i(t)\,dt > 0$, while $b_i(t)$ may be negative. It has been proved that if (H_1), ($\bar{H}_2$)–($\bar{H}_4$) hold, and

($\bar{H}_8$) there exist strictly positive constants s_i, θ_j such that

$$a_{ii}(t) > \sum_{k=1,k\neq i}^{n} a_{ki}(t) + \sum_{l=1}^{m} d_{li}(t),$$
$$e_{jj}(t) > \sum_{k=1}^{n} c_{kj}(t) + \sum_{l=1,l\neq j}^{m} e_{lj}(t),$$

then there exist a unique globally attractive positive T-periodic solution. Clearly, (H_8) cannot be inferred from ($\bar{H}_8$). Hence, Theorem 5.2 is fresh on the existence of globally attractive positive (componentwise) periodic solution and (H_8) is more general, which improve the main results in [19,20].

APPLICATION 5.3. Let us consider the Lotka-Volterra competition system by "pure-delay-type" which has been studied in [13]:

$$\dot{x}_i(t) = x_i(t) \left[b_i(t) - \sum_{k=1}^{n} a_{ik}(t) x_k(t - \tau_{ik}) \right], \qquad i = 1, \dots, n, \tag{5.1}$$

where $b_i, a_{ik} \in C(R, [0, +\infty))$ are T-periodic for some common periodic $T > 0$.

Applying our main results to (5.1), we reach the following conclusions.

THEOREM 5.3. *In addition to* (H_1), *if system (5.1) satisfies*

$$(\mathrm{H}_9) \qquad \int_0^T \left[b_i(t) - \sum_{k=1,k\neq i}^{n} a_{ik}(t+\tau_{ik}) x_k^*(t) \right] dt > 0,$$

and

$$B_i(t+\tau_{ii}) a_{ii}(t+\tau_{ii}) \exp\left\{ b_i^0(t,\tau_{ii}) \right\} < a_{ii}(t),$$

and $\bar{P}(t) = (f_{ik}(t))_{n\times n}$ *is a generalized uniform* M*-matrix, where* $f_{ik}(t)$ *is defined in Theorem 3.1, then system (5.1) has a unique globally attractive positive* T*-periodic solution.*

Similar to Corollary 3.1, one deduces the following.

COROLLARY 5.1. *In addition to* (H_1), (H_9), *if system (5.1) satisfies*

$$p_i \bar{B}_{ii}(t,\tau_{ii}) a_{ii}(t+\tau_{ii}) \exp\left\{ b_i^0(t,\tau_{ii}) \right\} < a_{ii}(t)$$

and $\bar{G}(t) = (\bar{f}_{ik}(t))_{n\times n}$ *is an eventually uniform* M*-matrix, where* $\bar{f}_{ik}(t)$ *is defined in Corollary 3.1, then system (5.1) has a unique globally attractive positive* T*-periodic solution.*

REMARK 5.3. Gopalsamy and He's sufficient conditions for the globally attractive periodic solution of (5.1) are of the form

$$b_i^{\iota} > 0, \qquad a_{ii}^{\iota} > 0, \qquad b_i^{\iota} > \sum_{k=1,k\neq i}^{n} a_{ik}^{\mu} p_k,$$
$$p_i \bar{B}_{ii}(t,\tau_{ii}) a_{ii}(t+\tau_{ii}) \exp\left\{ b_i^0(t,\tau_{ii}) \right\} < a_{ii}^{\iota},$$

and $\bar{P}(t) = (\bar{f}_{ik}(t))_{n\times n}$ is an eventually uniform M-matrix, where $\bar{f}_{ik}(t)$ is defined in Corollary 3.1.

Obviously, Theorem 5.3 and Corollary 5.1 improve the results in [13]. When $n = 2$, we also improve the results in [12].

APPLICATION 5.4. Now we consider the Lotka-Volterra competition system which was studied in [11]:

$$(5.2) \qquad \dot{x}_i(t) = x_i(t) \left[b_i(t) - \sum_{k=1}^{n} a_{ik}(t) x_k(t) \right], \qquad i = 1,\ldots,n,$$

where $b_i, a_{ik} \in C(R,[0,+\infty))$ are T-periodic for some common periodic $T > 0$. Note that when $n = 1$, (5.2) is the famous logistic equation. Our main results are valid to this special case.

THEOREM 5.4. *In addition to* (H_1), *if system (5.2) satisfies*

$$(\mathrm{H}_{10}) \qquad \int_0^T \left[b_i(t) - \sum_{k=1,k\neq i}^{n} a_{ik}(t) x_k^*(t) \right] dt > 0,$$

(H_{11}) *there exist strictly positive constants* s_i *and positive* T*-periodic functions* $\lambda_i(t)$ *and* $\mu_j(t)$ *such that*

$$s_i a_{ii}(t) > \sum_{k=1,k\neq i}^{n} s_k a_{ki}(t) + \lambda_i(t),$$

then system (5.2) has a unique positive T*-periodic solution which is globally attractive.*

REMARK 5.4. For more detail corollaries about system (5.2), one can refer to [29]. As pointed out in [29], our results improve those in [6,7,9–11].

6. EXAMPLE

EXAMPLE 6.1. Now we consider a two-species competition "pure-delay type" system of the form

$$\begin{aligned} \dot{x}_1 &= x_1\left[\frac{5}{2}e^{1/10} + e^{1/10}|\sin t| - 2x_1\left(t-\frac{1}{100}\right) - \frac{1}{2}x_2\left(t-\frac{1}{100}\right)\right], \\ \dot{x}_2 &= x_2\left[9 + |\sin t| - \frac{1}{8}(1+|\sin t|)x_1\left(t-\frac{1}{100}\right) - 2x_2\left(t-\frac{1}{100}\right)\right]. \end{aligned} \tag{6.1}$$

Corresponding to system (6.1), we have $b_1(t) = (5/2)e^{1/10} + e^{1/10}|\sin t|$, $b_2(t) = 9 + |\sin t|$, $a_{11}(t) = a_{22}(t) = 2$, $a_{12}(t) = 1/2$, $a_{21}(t) = (1/8)(1+|\sin t|)$, $\tau_{ik}(t) = 1/100$, $i,k = 1,2$.

Since $b_1^L = (5/2)e^{1/10}$ and $a_{12}^\mu(b_2^\mu/a_{22}^L)\exp\{b_2^\mu\tau_{22}\} = (5/2)e^{1/10}$, this cannot satisfy the conditions ($b_1^L > a_{12}^\mu(b_2^\mu/a_{22}^L)\exp\{b_2^\mu\tau_{22}\}$) in [13]. But, it is easy to verify that our result is feasible on (6.1). In fact, we know

$$\dot{x}_1 = x_1\left[\frac{5}{2}e^{1/10} + e^{1/10}|\sin t| - 2\exp\left\{-\frac{7}{2}e^{1/10}\times\frac{1}{100}\right\}x_1\right]$$

has a globally attractive unique π-periodic solution which can be represented as follows:

$$x_1^*(t) = \left[\int_{-\infty}^{t}\exp\left\{-\int_s^t\left[\frac{5}{2}e^{1/10} + e^{1/10}|\sin u|\right]du\right\}2e^{-(7/200)e^{1/10}}\,ds\right]^{-1}.$$

By amplifying, it follows that $x_1^*(t) \le (7/4)e^{1/10+(7/200)e^{1/10}} \le (7/4)e^{1/5}$. This leads to

$$\begin{aligned} m\left(b_2(t) - a_{21}\left(t+\frac{1}{100}\right)x_1^*(t)\right) &\ge m\left(9 + |\sin t| - \frac{1}{2}\times\frac{7}{4}e^{1/5}\right) \\ &\ge m(|\sin t|) > 0. \end{aligned}$$

Similarly, one knows

$$\dot{x}_2 = x_2\left[9 + |\sin t| - 2\exp\left\{-10\times\frac{1}{100}\right\}x_2\right]$$

has a globally attractive unique π-periodic solution which can be represented as follows:

$$x_2^*(t) = \left[\int_{-\infty}^{t}\exp\left\{-\int_s^t[9 + |\sin u|]\,du\right\}2e^{-1/10}\,ds\right]^{-1}.$$

By amplifying, it follows that $x_2^*(t) \le 5e^{1/10}$. This leads to

$$\begin{aligned} m\left(b_1(t) - a_{12}\left(t+\frac{1}{100}\right)x_2^*(t)\right) &\ge m\left(\frac{5}{2}e^{1/10} + e^{1/10}|\sin t| - \frac{1}{2}\times\frac{5}{2}e^{1/10}\right) \\ &= m(|\sin t|) > 0. \end{aligned}$$

Through simple calculation and by Corollary 5.1, system (6.1) has a globally attractive π-periodic solution.

7. CONCLUSION

In this paper, we study a multispecies predator-prey system by "pure-delay type", in which the competition among predator species and among prey species is simultaneously considered. A set of sufficient conditions have been obtained for the existence and uniqueness of periodic solution which is globally attractive. Moreover, in this paper, we introduce a new concept of *generalized uniform M-matrix*. As we can see, by using the comparison theorem and generalizing the concept of *eventually uniform M-matrix* to *generalized uniform M-matrix*, the results which guarantee the existence and uniqueness of periodic solution generalize and improve the results in [6,7, 9–13,19,20,22]. It is in this sense, our new concept and results are the best. But we note that it is not easy to verify the conditions if the stabilizing negative feedback coefficients are truly nonconstant and periodic.

REFERENCES

1. L. Chen, *Mathematical Models and Methods in Ecology*, (in Chinese), Science Press, Beijing, (1988).
2. Y. Kuang, *Delay Differential Equations with Application in Population Dynamics*, New York, Academic Press, (1988).
3. B. Tang and Y. Kuang, Existence, uniqueness and asymptotic stability of periodic solutions of periodic functional differential systems, *Tohoku Math. J.* **49**, 217–239, (1997).
4. P. Korman, Some new results on the periodic competition model, *J. Math. Anal. Appl.* **171**, 131–138, (1992).
5. J.C. Eilbeck and J. Lopez-Gomez, On the periodic Lotka-Volterra competition model, *J. Math. Anal. Appl.* **210**, 58–87, (1997).
6. M. Araki and M. Kondo, Stability and transient behavior of composite systems, *IEEE Trans. Automat. Control* **17**, 537–541, (1972).
7. X. Zhao, The qualitative analysis of n-species Lotka-Volterra periodic competition systems, *Mathl. Comput. Modelling* **15** (11), 3–8, (1991).
8. J.M. Cushing, Two species competition in a periodic environment, *J. Math. Biol.* **24**, 381–403, (1986).
9. S. Ahmad, Convergence and ultimate bounds of solutions of nonautonomous Volterra-Lotka competition equations, *J. Math. Anal. Appl.* **127**, 377–387, (1987).
10. K. Gopalsamy, Exchange of equilibria in two species Lotka-Volterra competition models, *J. Austral. Math. Soc. Ser. B* **24**, 160–170, (1982).
11. K. Gopalsamy, Global asymptotic stability in a periodic Lotka-Volterra system, *J. Austral. Math. Soc. Ser. B* **27**, 66–72, (1985).
12. C. Alvarz. and A.C. Lazer, An application of topological degree to the periodic competing species problem, *J. Austral. Math. Soc. Ser. B* **28**, 202–219, (1986).
13. K. Gopalsamy and X.Z. He, Global stability in n-species competition modelled by "pure-delay type" system—II: Nonautonomous case, *Canadian Applied Mathematics Quarterly* **6** (1), 17–43, (1998).
14. X.Z. He and K. Gopalsamy, Dynamics of Lotka-Volterra mutualism in changing environments, *Nonlinear World* **1**, 173–185, (1994).
15. W.X. Ma and K.I. Maruno, Complexiton solutions of the Toda lattice equation, *Phys. A* **343**, 219–237, (2004).
16. Z. Ma and W. Wang, Asymptotic behavior of predator-prey system with time dependent coefficients, *Appl. Anal.* **34**, 79–90, (1989).
17. E. Beretta and Y. Kuang, Convergence results in a well-known delayed predator-prey system, *J. Math. Anal. Appl.* **204**, 840–853, (1996).
18. J. Lopez-Gomez, R. Ortega and A. Tineo, The periodic predator-prey Lotka-Volterra model, *Advances in Differential Equations* **1**, 403–423, (1996).
19. P. Yang and R. Xu, Global attractivity of the periodic Lotka-Volterra system, *J. Math. Anal. Appl.* **233**, 221–232, (1999).
20. J. Zhao and W. Chen, Global asymptotic stability of a periodic ecological model, *Appl. Math. Comput.* **147**, 881–892, (2004).
21. Y.H. Xia, F. Chen, A. Chen and J. Cao, Existence and global attractivity of an almost periodic ecological model, *Appl. Math. Comput.* **157** (2), 449–475, (2004).
22. X. Wen, Global attractivity of positive solution of multispecies ecological competition-predator delay system, *Acta Mathematica Sinica* **45** (1), 83–92, (2002).
23. T. Burton, *Stability and Periodic Solution of Ordinary and Functional Differential Equations*, Academic Press, New York, (1985).
24. J.K. Hale and S.M. Verduyn Lunel, *Introduction to Functional Differential Equations*, Springer-Verlag, New York, (1993).
25. D. DeAngelis, W.M. Post and C.C. Travis, *Positive Feedback in Natural System*, Springer-Verlag, New York, (1986).
26. B.S. Goh, Stability in models of mutualism, *Amer. Natural* **113**, 261–275.
27. J. Hofbauer and K. Sigmund, *Dynamical Systems and the Theory of Evolution*, Cambridge Univ. Press, Cambridge, England.
28. T. Yoshizawa, Stability theory for the existence of periodic solutions and almost-periodic solutions, In *Applied Mathematical Sciences, Volume 14*, Springer-Verlag, New York, (1975).
29. Y.H. Xia, J. Cao, H. Zhang and F. Chen, Almost periodic solutions of n-species competitive system with feedback controls, *J. Math. Anal. Appl.* **294** (2), 503–522, (2004).